CHAPTER 7

Converting Among Effect Sizes

INTRODUCTION

Earlier in this Part we discussed the case where different study designs were used to compute the same effect size. For example, studies that used independent groups and studies that used matched groups were both used to yield estimates of the standardized mean difference, g. There is no problem in combining these estimates in a meta-analysis since the effect size has the same meaning in all studies.

Consider, however, the case where some studies report a difference in means, which is used to compute a standardized mean difference. Others report a difference in proportions which is used to compute an odds ratio. And others report a correlation. All the studies address the same broad question, and we want to include them in one meta-analysis. Unlike the earlier case, we are now dealing with different indices, and we need to convert them to a common index before we can proceed.

The question of whether or not it is appropriate to combine effect sizes from studies that used different metrics must be considered on a case by case basis. The key issue is that it only makes sense to compute a summary effect from studies that we judge to be comparable in relevant ways. If we would be comfortable combining these studies if they had used the same metric, then the fact that they used different metrics should not be an impediment.

For example, suppose that several randomized controlled trials start with the same measure, on a continuous scale, but some report the outcome as a mean and others dichotomize the outcome and report it as success or failure. In this case, it may be highly appropriate to transform the standardized mean differences

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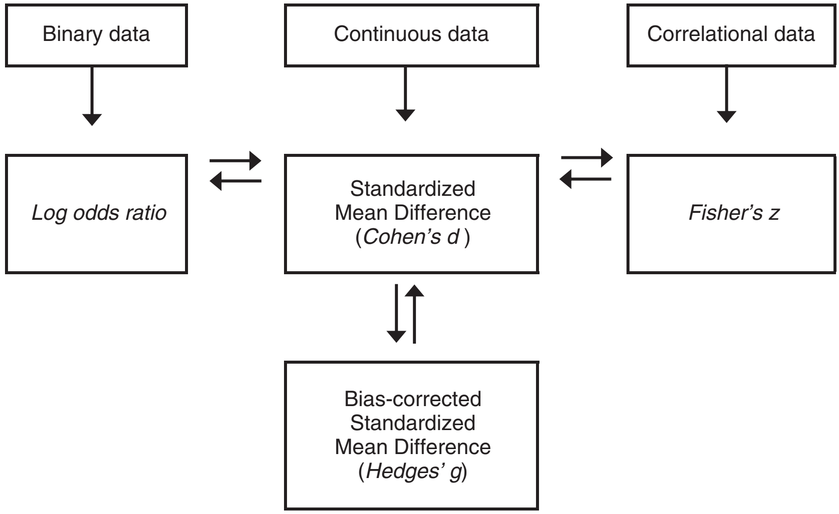

Figure 7.1 Converting among effect sizes.

and the odds ratios to a common metric and then combine them across studies. By contrast, observational studies that report correlations may be substantially different from observational studies that report odds ratios. In this case, even if there is no technical barrier to converting the effects to a common metric, it may be a bad idea from a substantive perspective.

In this chapter we present formulas for converting between an odds ratio and *d*, or between *d* and *r*. By combining formulas it is also possible to convert from an odds ratio, via *d*, to *r* (see Figure 7.1). In every case the formula for converting the effect size is accompanied by a formula to convert the variance.

When we convert between different measures we make certain assumptions about the nature of the underlying traits or effects. Even if these assumptions do not hold exactly, the decision to use these conversions is often better than the alternative, which is to simply omit the studies that happened to use an alternate metric. This would involve loss of information, and possibly the *systematic* loss of information, resulting in a biased sample of studies. A sensitivity analysis to compare the meta-analysis results with and without the converted studies would be important.

Figure 7.1 outlines the mechanism for incorporating multiple kinds of data in the same meta-analysis. First, each study is used to compute an effect size and variance of its *native* index, the log odds ratio for binary data, *d* for continuous data, and *r* for correlational data. Then, we convert all of these indices to a common index, which would be either the log odds ratio, *d*, or *r*. If the final index is *d*, we can move from there to Hedges' *g*. This common index and its variance are then used in the analysis.

CONVERTING FROM THE LOG ODDS RATIO TO *d*

We can convert from a log odds ratio (*LogOddsRatio*) to the standardized mean difference *d* using

$$d = LogOddsRatio \times \frac{\sqrt{3}}{\pi}, \tag{7.1}$$

where π is the mathematical constant (approximately 3.14159). The variance of *d* would then be

$$V_d = V_{LogOddsRatio} \times \frac{3}{\pi^2}, \tag{7.2}$$

where $V_{LogOddsRatio}$ is the variance of the log odds ratio. This method was originally proposed by Hasselblad and Hedges (1995) but variations have been proposed (see Sanchez-Meca, Marin-Martinez, & Chacon-Moscoso, 2003; Whitehead, 2002). It assumes that an underlying continuous trait exists and has a logistic distribution (which is similar to a normal distribution) in each group. In practice, it will be difficult to test this assumption.

For example, if the log odds ratio were $LogOddsRatio = 0.9069$ with a variance of $V_{LogOddsRatio} = 0.0676$, then

$$d = 0.9069 \times \frac{\sqrt{3}}{3.1416} = 0.5000$$

with variance

$$V_d = 0.0676 \times \frac{3}{3.1416^2} = 0.0205.$$

CONVERTING FROM *d* to the log odds ratio

We can convert from the standardized mean difference *d* to the log odds ratio (*LogOddsRatio*) using

$$LogOddsRatio = d\frac{\pi}{\sqrt{3}}, \tag{7.3}$$

where π is the mathematical constant (approximately 3.14159). The variance of *LogOddsRatio* would then be

$$V_{LogOddsRatio} = V_d\frac{\pi^2}{3}. \tag{7.4}$$

For example, if $d = 0.5000$ and $V_d = 0.0205$ then

$$LogOddsRatio = 0.5000 \times \frac{3.1416}{\sqrt{3}} = 0.9069,$$

and

$$V_{LogOddsRatio} = 0.0205 \times \frac{3.1416^2}{3} = 0.0676.$$

To employ this transformation we assume that the continuous data have the logistic distribution.

CONVERTING FROM *r* TO *d*

We convert from a correlation (r) to a standardized mean difference (d) using

$$d = \frac{2r}{\sqrt{1 - r^2}}. \tag{7.5}$$

The variance of d computed in this way (converted from r) is

$$V_d = \frac{4V_r}{(1 - r^2)^3}. \tag{7.6}$$

For example, if $r = 0.50$ and $V_r = 0.0058$, then

$$d = \frac{2 \times 0.50}{\sqrt{1 - 0.50^2}} = 1.1547$$

and the variance of d is

$$V_d = \frac{4 \times 0.0058}{\left(1 - 0.50^2\right)^3} = 0.0550.$$

In applying this conversion we assume that the continuous data used to compute r has a bivariate normal distribution and that the two groups are created by dichotomizing one of the two variables.

CONVERTING FROM *d* TO *r*

We can convert from a standardized mean difference (d) to a correlation (r) using

$$r = \frac{d}{\sqrt{d^2 + a}} \tag{7.7}$$

where a is a correction factor for cases where $n_1 \neq n_2$,

$$a = \frac{(n_1 + n_2)^2}{n_1 n_2}. \tag{7.8}$$

The correction factor (a) depends on the ratio of n_1 to n_2, rather than the absolute values of these numbers. Therefore, if n_1 and n_2 are not known precisely, use $n_1 = n_2$, which will yield $a = 4$. The variance of r computed in this way (converted from d) is

$$V_r = \frac{a^2 V_d}{(d^2 + a)^3}. \tag{7.9}$$

For example, if $n_1 = n_2$, $d = 1.1547$ and $v_d = 0.0550$, then

$$r = \frac{1.1547}{\sqrt{1.1547^2 + 4}} = 0.5000$$

and the variance of r converted from d will be

$$V_r = \frac{4^2 \times 0.0550}{\left(1.1547^2 + 4\right)^3} = 0.0058.$$

In applying this conversion assume that a continuous variable was dichotomized to create the treatment and control groups.

When we transform between Fisher's z and d we are making assumptions about the independent variable only. When we transform between the log odds ratio and d we are making assumptions about the dependent variable only. As such, the two sets of assumptions are independent of each other, and one has no implications for the validity of the other. Therefore, we can apply both sets of assumptions and transform from Fisher's z through d to the log odds ratio, as well as the reverse.

SUMMARY POINTS

- If all studies in the analysis are based on the same kind of data (means, binary, or correlational), the researcher should select an effect size based on that kind of data.
- When some studies use means, others use binary data, and others use correlational data, we can apply formulas to convert among effect sizes.
- Studies that used different measures may differ from each other in substantive ways, and we need to consider this possibility when deciding if it makes sense to include the various studies in the same analysis.